DSP

محاضرة ② د. محمد عرفة الخميس 8/10/2015

#Classification of discrete time systems :-

[1] Static and dynamic discrete-time systems

- static ⇒ o/p of the system depends upon the present i/p only. (memoryless)
- dynamic ⇒ o/p of the system depends upon the past i/p also. (memory)

Ex : $y(n) = 10\, x(n)$ ⟹ static ⇒ causal

$y(n) = 15x^2(n) + \cos(x(n))$ ⟹ static ⇒ causal

$y(n) = x(n-1) + 3x(n)^2$ ⟹ dynamic ⇒ causal

$y(n) = x(n+1) + x(n) + x(n-1)$ ⟹ dynamic ⇒ non causal

---

[2] Causal and non causal system

Causal system ⇒ o/p depends on the present and past i/p

non Causal System ⇒ o/p depends on the future i/p also.
(doesn't exist in reality "unreal")

Ex : $y(n) = x(2n)$

$n=0 \Rightarrow y(0) = x(0)$

$n=1 \Rightarrow y(1) = x(2)$

⟹ non causal

the same concept works for $y(n) = x(n^2)$

3 Shift variant system - shift invariant syste...

≡ time ⁄ ⁄ - time ⁄ ⁄

time invariant System ⇒ o/p chs. don't change with shifting in time

time variant System ⇒ o/p chs. change with shifting in time

* for discrete time system

$y(n) = T[x(n)]$

Shifting by k

$y(n,k) = T[x(n-k)]$

$y(n-k) = y(n)\Big|_{n \to n-k}$

⇒ time invariant System.

if $y(n,k) \neq y(n-k)$ ⇒ time variant system

Ex: 1- $y(n) = x(n)\cos(\omega n)$

2- $y(n) = x(n) + x(n-2)$

3- $y(n) = x(-n)$

Solution

1 $y(n,k) = x(n-k)\cos(\omega n)$

$y(n-k) = y(n)\Big|_{n \to n-k} = x(n-k)\cos(\omega(n-k))$

$y(n,k) \neq y(n-k)$

⇒ Time variant System

(2) $y(n,k) = x(n-k) + x(n-2-k)$

$y(n-k) = y(n)\Big|_{n \to n-k} = x(n-k) + x(n-k-2)$

$y(n,k) = y(n-k) \Rightarrow$ time invariant system

(3) $y(n) = x(-n)$

$y(n,k) = x(-n-k)$

$y(n,k) = y(n)\Big|_{n \to n-k} = x(-(n-k)) = x(-n+k)$

$y(n,k) \neq y(n-k) \Rightarrow$ time variant system

Ex: $y(n) = x(n^2)$

$y(n,k) = x(n^2-k)$

$y(n-k) = y(n)\Big|_{n \to n-k} = x((n-k)^2)$

$y(n,k) \neq y(n-k)$

$\Rightarrow$ time variant system

---

[4] linear and non-linear system

$x_1(n) \rightarrow [T] \rightarrow y_1(n) = T[x_1(n)]$

$x_2(n) \rightarrow [T] \rightarrow y_2(n) = T[x_2(n)]$

$x_3(n) \rightarrow [T] \rightarrow y_3(n) = T[x_3(n)]$

if $y_3(n) = y_1(n) + y_2(n) \Rightarrow$ linear system

if $y_3(n) \neq y_1(n) + y_2(n) \Rightarrow$ non-linear system

Ex:- 1- $y(n) = x(n^2)$

2- $y(n) = x^2(n)$

3- $y(n) = \log(x(n))$

Solution

1] $y(n) = x(n^2)$

$y_1(n) = x_1(n^2)$

$y_2(n) = x_2(n^2)$

$y_3(n) = x_1(n^2) + x_2(n^2)$

$y_3(n) = y_1(n) + y_2(n)$

$\Rightarrow$ linear system

2] $y(n) = x^2(n)$

$y_1(n) = x_1^2(n)$

$y_2(n) = x_2^2(n)$

$y_3(n) = (x_1(n) + x_2(n))^2$

$y_3(n) \neq y_1(n) + y_2(n)$

$\Rightarrow$ non linear system

3] $y(n) = \log(x(n)) \Rightarrow$ non linear system

4] $y(n) = \cos(x(n)) \Rightarrow$ non linear system

5 Stable and unstable system.

Stable ⇒ for every bounded i/p sequence, ther exist bounded o/p sequence.

(BIBO)

Ex:

1- $y(n) = x(n^2)$ ⇒ stable

2- $y(n) = x^2(n)$ ⇒ stable

3- $y(n) = 2^n x(n)$ ⇒ unstable

4- $y(n) = x(n)\cos(\omega n)$ ⇒ stable

5- $y(n) = nx(n)$ ⇒ unstable

Ex: Check all characteristics

① $y(n) = x(-n+2)$

1- Dynamic

2- non causal (future i/p) $y(0) = x(2)$

3- $y(n,k) = x(-n+2-k)$

$y(n-k) = y(n)\big|_{n \to n-k}$

$= x(-(n-k)+2) = x(-n+k+2)$

$y(n,k) \neq y(n-k)$ ⇒ time variant system

4- linear

5- stable

② $y(n) = x(n) + nx(n+1)$

1- Dynamic

2- non causal ($x(n+1)$ future i/p)

3- $y(n,k) = x(n-k) + n\,x(n+1-k)$

$y(n-k) = y(n)\big|_{n \to n-k}$

$y(n-k) = x(n-k) + (n-k)\,x(n+1-k)$

$y(n,k) \neq y(n-k)$ ⇒ time variant

4- linear

5- unstable

---

③ $y(n) = \cos(x(n))$

1- static.

2- Causal.

3- time invariant.

4- Non linear

5- stable

report:

1- $y(n) = \sum_{k=-a}^{n+1} x(k)$

2- $y(n) = x(2n)$

3- $y(n) = x(-n)$

تسلم في بيكشن - نفس إجابة الأخيرة

Linear Convolution:

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

(only when $k=n$)

any discrete time signal can be expressed in terms of unit sample sequence (unit impulse)

for a discrete time system

$x(n) \rightarrow \boxed{T} \rightarrow y(n) = T[x(n)]$

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right]$$

The System is assumed to be linear time invariant system (LTI)

$$y(n) = T[\cdots + x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + \cdots]$$

as the system is linear

$$y(n) = \cdots + T[x(-2)\,\delta(n+2)] + T[x(-1)\,\delta(n+1)] + T[x(0)\,\delta(n)] + T[x(1)\,\delta(n-1)] + \cdots$$

Constant value

$$y(n) = \cdots + x(-2)\,T[\delta(n+2)] + x(-1)\,T[\delta(n+1)] + x(0)\,T[\delta(n)] + x(1)\,T[\delta(n-1)] + \cdots$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,T[\delta(n-k)]$$

⇒ unit impulse response

$$h(n) = T[\delta(n)]$$

$$h(n-k) = T[\delta(n-k)]$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Linear Convolution

* The system response to any input sequence $x(n)$, can be calculated by the known of unit impulse response $h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$x(n)$ ⇒ the i/p sequence

$y(n)$ ⇒ the required response for the i/p $x(n)$

$h(n)$ ⇒ the known unit impulse response

$$h(n) = y(n) \Big|_{x(n)=\delta(n)}$$

⇒ Turn over

Ex: $x(n) = \{1, 1, 1, 1\}$

$h(n) = \{2, 2\}$

Compute the Convolution of the two sequences

Solution

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$n_{start} = n_{x_{start}} + n_{h_{start}}$$

$$= 0 + 0 = 0$$

$$n_{end} = n_{x_{end}} + n_{h_{end}}$$

$$= 3 + 1 = 4$$

at $\boxed{n=0} \Rightarrow y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$$= \sum_{k=0}^{3} x(k)\, h(n-k)$$

$$= x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3)$$

($x(2)h(-2)$ and $x(3)h(-3)$ are 0)

$\therefore y(0) = 2$

at $\boxed{n=1} \Rightarrow y(1) = \sum_{k=0}^{3} x(k)\, h(1-k)$

$$= x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2)$$

($x(2)h(-1)$ and $x(3)h(-2)$ are 0)

$$= 2 + 2 = 4$$

at $\boxed{n=2} \Rightarrow y(2) = \sum_{k=0}^{3} x(k)\, h(2-k)$

$$= x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(-1)$$

($x(3)h(-1)$ is 0)

$$= 2 + 2 = 4$$

at $\boxed{n=3} \Rightarrow y(3)=4$

at $\boxed{n=4} \Rightarrow y(4)=2$

$\therefore \quad y(n)=\{2,4,4,4,2\}$

Ex: $x(n)=\{1,1,\underset{\uparrow}{0},1,1\}$ Convolution

$h(n)=\{1,-2,-3,\underset{\uparrow}{4}\}$

Solution

$$y(n)=\sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$n_{start} = n_{x\,start} + n_{h\,start} = -2-3 = -5$

$n_{end} = n_{x\,end} + n_{h\,end} = 2+0 = 2$

$\boxed{n=-5} \Rightarrow y(-5)=\sum_{k=-2}^{2} x(k)\,h(n-k)$

$$y(-5)=\sum_{k=-2}^{2} x(k)\,h(-5-k)$$

$$= x(-2)h(-3)+x(-1)\underbrace{h(-4)}_{0}+x(0)\underbrace{h(-5)}_{0}+x(1)\underbrace{h(-6)}_{0}+x(2)\underbrace{h(-7)}_{0}$$

$$=(1)(1)=1$$

$\boxed{n=-4}$ $y(-4)=\sum_{k=-2}^{2} x(k)\,h(-4-k)$

$$= x(-2)h(-2)+x(-1)h(-3)+x(0)\underbrace{h(-4)}_{0}+x(1)\underbrace{h(-5)}_{0}+x(2)\underbrace{h(-6)}_{0}$$

$y(-4) = -2+1 = -1$

$n=-3 \Rightarrow y(-3) = -5$

$n=-2 \Rightarrow y(-2) = 2$

$n=-1 \Rightarrow y(-1) = 3$

$n=0 \Rightarrow y(0) = -5$

$n=1 \Rightarrow y(1) = 1$

$n=2 \Rightarrow y(2) = 4$

$$y(n) = x(n) * h(n) \quad \Leftarrow \text{Convolution}$$

$$= \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$